

4.3 Areas of Trapezoids

Essential Question How can you derive a formula for the area of a trapezoid?

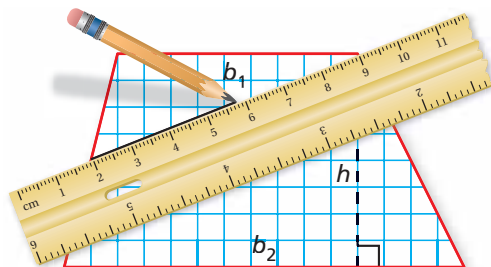
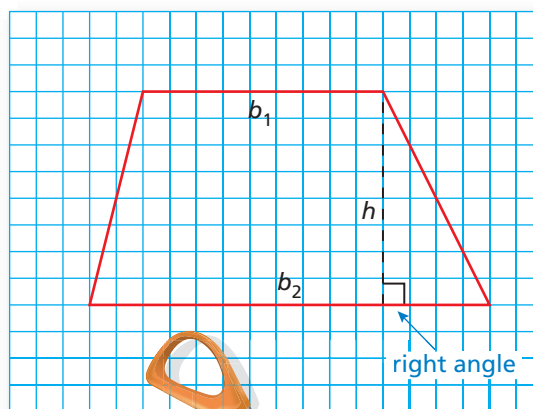
1 ACTIVITY: Deriving the Area Formula of a Trapezoid

Work with a partner. Use a piece of centimeter grid paper.

- Draw *any* trapezoid so that its base lies on one of the horizontal lines of the paper.
- Estimate the area of your trapezoid (in square centimeters) by counting unit squares.

Area \approx Estimate

- Label the height and the bases *inside* the trapezoid.
- Cut out the trapezoid. Mark the midpoint of the side opposite the height. Draw a line from the midpoint to the opposite upper vertex.
- Cut along the line. You will end up with a triangle and a quadrilateral. Arrange these two figures to form a figure whose area you know.

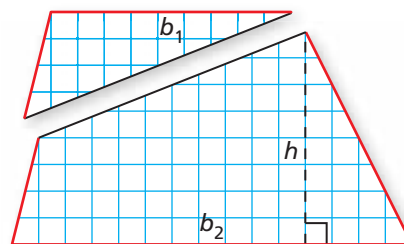


- Use your result to write a *formula* for the area of a trapezoid.

Area = Formula

- Use your formula to find the area of your trapezoid (in square centimeters).

Area = Exact Area



- Compare this area with your estimate in part (b).

Geometry

In this lesson, you will

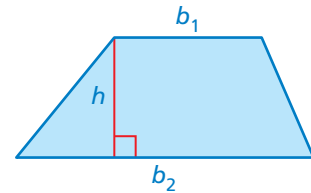
- find areas of trapezoids.
- solve real-life problems.

Key Idea

Area of a Trapezoid

Words The area A of a trapezoid is one-half the product of its height h and the sum of its bases b_1 and b_2 .

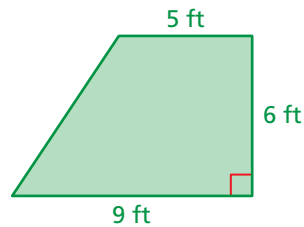
Algebra $A = \frac{1}{2}h(b_1 + b_2)$



EXAMPLE 1 Finding Areas of Trapezoids

Find the area of each trapezoid.

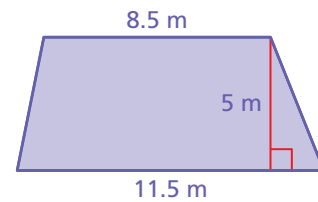
a.



$$\begin{aligned} A &= \frac{1}{2}h(b_1 + b_2) && \text{Write formula.} \\ &= \frac{1}{2}(6)(5 + 9) && \text{Substitute.} \\ &= \frac{1}{2}(6)(14) && \text{Add.} \\ &= 42 && \text{Multiply.} \end{aligned}$$

∴ The area of the trapezoid is 42 square feet.

b.



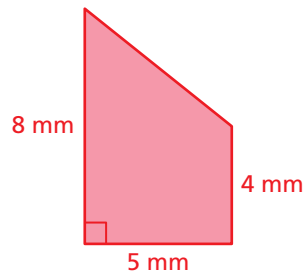
$$\begin{aligned} A &= \frac{1}{2}h(b_1 + b_2) \\ &= \frac{1}{2}(5)(8.5 + 11.5) \\ &= \frac{1}{2}(5)(20) \\ &= 50 \end{aligned}$$

∴ The area of the trapezoid is 50 square meters.

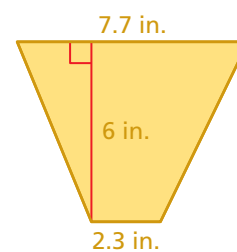
On Your Own

Find the area of the trapezoid.

1.

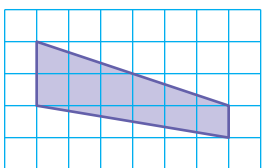


2.



Now You're Ready
Exercises 7–9

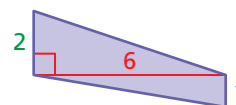
EXAMPLE 2 Finding the Area of a Trapezoid on a Grid



What is the area of the trapezoid?

- (A) 6 units² (B) 7 units² (C) 9 units² (D) 12 units²

Count grid lines to find the dimensions. The height h is 6 units, base b_1 is 1 unit, and base b_2 is 2 units.

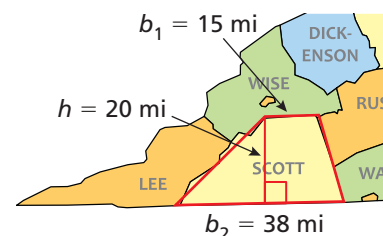


$$\begin{aligned}
 A &= \frac{1}{2}h(b_1 + b_2) && \text{Write formula.} \\
 &= \frac{1}{2}(6)(1 + 2) && \text{Substitute values.} \\
 &= \frac{1}{2}(6)(3) && \text{Add.} \\
 &= 9 && \text{Multiply.}
 \end{aligned}$$

- ∴ The area of the trapezoid is 9 square units. The correct answer is (C).

EXAMPLE 3 Real-Life Application

You can use a trapezoid to approximate the shape of Scott County, Virginia. The population is about 23,200. About how many people are there per square mile?



Find the area of Scott County.

$$\begin{aligned}
 A &= \frac{1}{2}h(b_1 + b_2) && \text{Write formula for area of a trapezoid.} \\
 &= \frac{1}{2}(20)(15 + 38) && \text{Substitute 20 for } h, 15 \text{ for } b_1, \text{ and } 38 \text{ for } b_2. \\
 &= \frac{1}{2}(20)(53) = 530 && \text{Simplify.}
 \end{aligned}$$

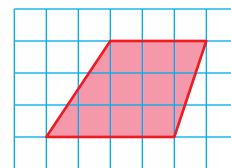
The area of Scott County is about 530 square miles. Divide the population by the area to find the number of people per square mile.

- ∴ So, there are about $\frac{23,200 \text{ people}}{530 \text{ mi}^2} \approx 44$ people per square mile.

On Your Own

Now You're Ready
Exercises 11–13

- Find the area of the trapezoid.
- WHAT IF?** In Example 3, the population of Scott County decreases by 550. By how much does the number of people per square mile change? Explain.



4.3 Exercises

Vocabulary and Concept Check

- VOCABULARY** Identify the bases and the height of the trapezoid.
- REASONING** What measures do you need to find the area of a trapezoid?
- WHICH ONE DOESN'T BELONG?** Which one does *not* belong with the other three? Explain your reasoning.



$$\frac{1}{2}bh$$

$$\ell w$$

$$2\ell + 2w$$

$$\frac{1}{2}h(b_1 + b_2)$$

Practice and Problem Solving

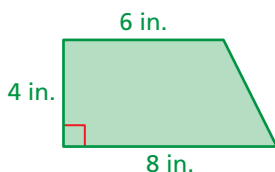
Find the area of the trapezoid.

4. $b_1 = 4, b_2 = 8, h = 2$

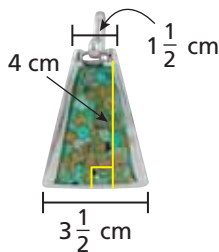
5. $b_1 = 5, b_2 = 7, h = 4$

6. $b_1 = 12, b_2 = 6, h = 3$

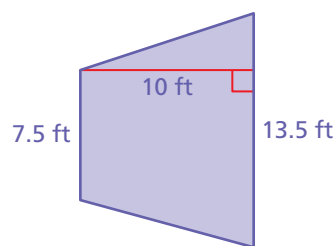
1 7.



8.



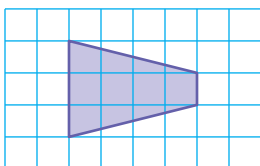
9.



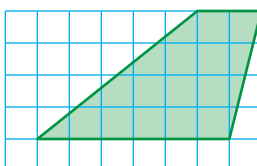
10. **ERROR ANALYSIS** Describe and correct the error in finding the area of the trapezoid.

Find the area of the trapezoid.

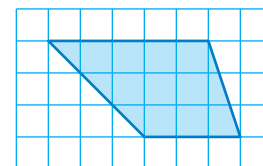
2 11.



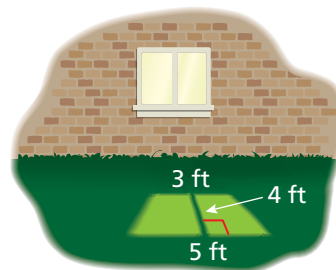
12.



13.



14. **LIGHT** Light shines through a window. What is the area of the trapezoid-shaped region created by the light?



Find the area of a trapezoid with height h and bases b_1 and b_2 .

15. $h = 6$ in.

$b_1 = 9$ in.

$b_2 = 11$ in.

16. $h = 22$ cm

$b_1 = 10.5$ cm

$b_2 = 12.5$ cm

17. $h = 12$ mi

$b_1 = 5.6$ mi

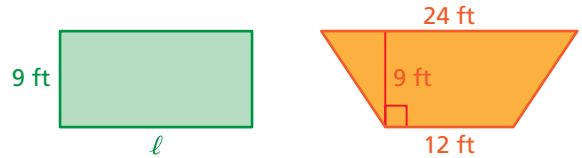
$b_2 = 7.4$ mi

18. $h = 14$ m

$b_1 = 21$ m

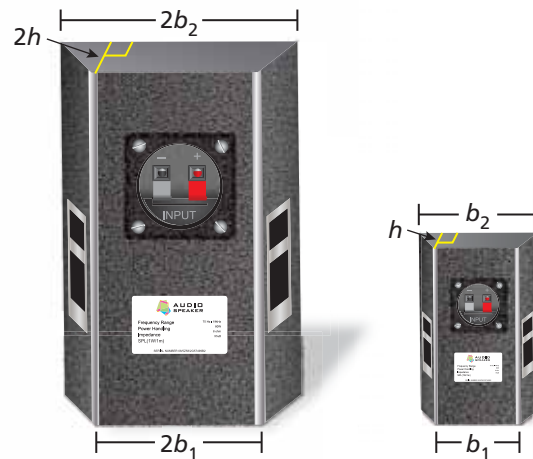
$b_2 = 22$ m

19. **REASONING** The rectangle and the trapezoid have the same area. What is the length ℓ of the rectangle?



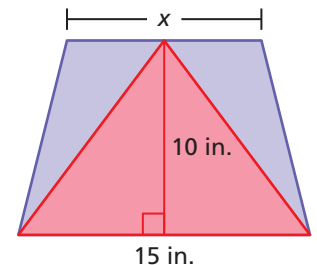
20. **OPEN-ENDED** The area of the trapezoidal student election sign is 5 square feet. Find two possible values for each base length.

21. **AUDIO** How many times greater is the area of the floor covered by the larger speaker than by the smaller speaker?



22. **Critical Thinking** The triangle and the trapezoid share a 15-inch base and a height of 10 inches.

- The area of the trapezoid is less than twice the area of the triangle. Find the values of x . Explain your reasoning.
- Can the area of the trapezoid be exactly twice the area of the triangle? Explain your reasoning.



Fair Game Review what you learned in previous grades & lessons

Plot the ordered pair in a coordinate plane. *(Skills Review Handbook)*

23. $(5, 0)$

24. $(2, 4)$

25. $(0, 3)$

26. $(6, 1)$

27. **MULTIPLE CHOICE** Which expression represents “6 more than x ”? *(Section 3.2)*

(A) $6 - x$

(B) $6x$

(C) $x + 6$

(D) $\frac{6}{x}$