

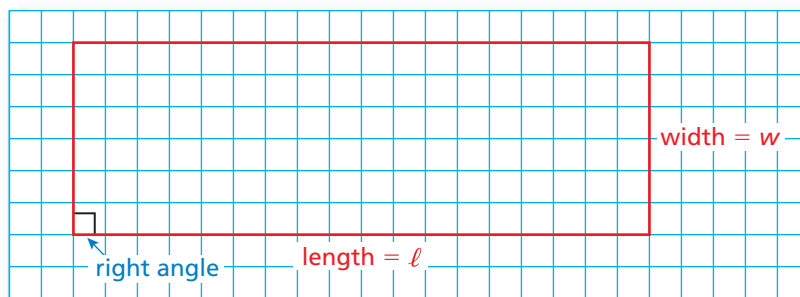
## 4.2 Areas of Triangles

**Essential Question** How can you derive a formula for the area of a triangle?

### 1 ACTIVITY: Deriving the Area Formula of a Triangle

Work with a partner.

- a. Draw *any* rectangle on a piece of grid paper. An example is shown below. Label the length and width. Then find the area of your rectangle.



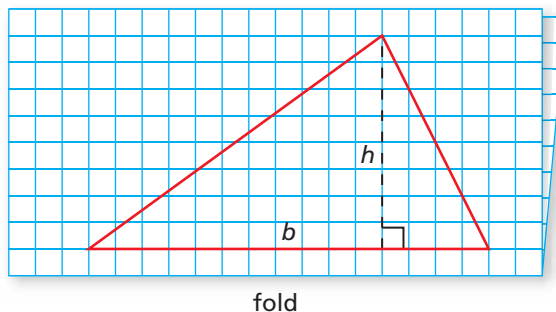
- b. Draw a diagonal from one corner of your rectangle to the opposite corner. Cut along the diagonal. Compare the area of the rectangle with the area of the two pieces you cut. What do you notice? Use your results to write a formula for the area  $A$  of a triangle.

$A =$   Formula

### 2 ACTIVITY: Deriving the Area Formula of a Triangle

Work with a partner.

- a. Fold a piece of grid paper in half. Draw a triangle so that its base lies on one of the horizontal lines of the paper. Do not use a right triangle. Label the height and the base *inside* the triangle.



- b. Estimate the area of your triangle by counting unit squares. Area  $\approx$   Estimate
- c. Cut out the triangle so that you end up with two identical triangles. Form a quadrilateral whose area you know. What type of quadrilateral is it? Explain how you *know* it is this type.
- d. Use your results to write a formula for the area of a triangle. Then use your formula to find the exact area of your triangle. Compare this area with your estimate in part (b).

#### Geometry

In this lesson, you will

- find areas of triangles.
- solve real-life problems.

## Math Practice

### Calculate Accurately

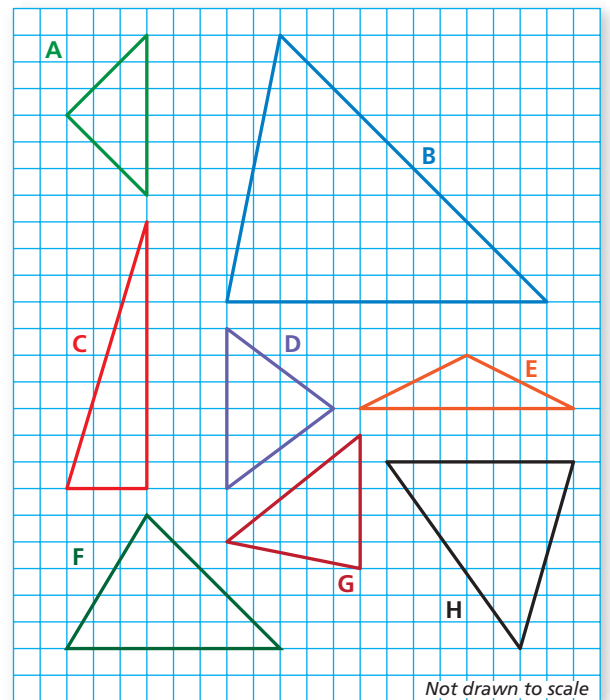
How can you estimate the area of each triangle so that the answer is close to the exact area?

## 3 ACTIVITY: Estimating and Finding the Area of a Triangle

Work with a partner. Each grid square represents 1 square centimeter.

- Use estimation to match each triangle with its area.
- Then check your work by finding the exact area of each triangle.

	Area	Estimate Match	Exact Match
a.	$15 \text{ cm}^2$	<input type="checkbox"/>	<input type="checkbox"/>
b.	$20 \text{ cm}^2$	<input type="checkbox"/>	<input type="checkbox"/>
c.	$9 \text{ cm}^2$	<input type="checkbox"/>	<input type="checkbox"/>
d.	$12 \text{ cm}^2$	<input type="checkbox"/>	<input type="checkbox"/>
e.	$60 \text{ cm}^2$	<input type="checkbox"/>	<input type="checkbox"/>
f.	$12\frac{1}{2} \text{ cm}^2$	<input type="checkbox"/>	<input type="checkbox"/>
g.	$24\frac{1}{2} \text{ cm}^2$	<input type="checkbox"/>	<input type="checkbox"/>
h.	$8 \text{ cm}^2$	<input type="checkbox"/>	<input type="checkbox"/>



## What Is Your Answer?

4. **PARTNER ACTIVITY** Use a piece of centimeter grid paper to create your own “triangle matching activity.” Trade with your partner and solve each other’s matching activity.
5. **IN YOUR OWN WORDS** How can you derive a formula for the area of a triangle?

### Practice

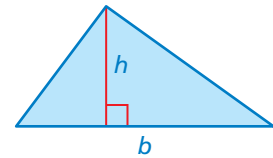
Use what you learned about the areas of triangles to complete Exercises 3–5 on page 162.

## Key Idea

### Area of a Triangle

**Words** The area  $A$  of a triangle is one-half the product of its base  $b$  and its height  $h$ .

**Algebra**  $A = \frac{1}{2}bh$



## EXAMPLE 1 Finding the Area of a Triangle

Find the area of the triangle.

$$A = \frac{1}{2}bh$$

Write formula.

$$= \frac{1}{2}(5)(8)$$

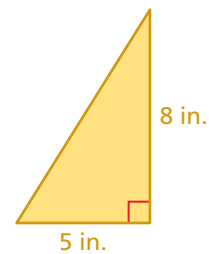
Substitute 5 for  $b$  and 8 for  $h$ .

$$= \frac{1}{2}(40)$$

Multiply 5 and 8.

$$= 20$$

Multiply  $\frac{1}{2}$  and 40.

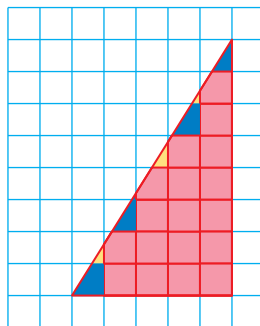


### Remember

In Example 1, use the Associative Property of Multiplication to multiply 5 and 8 first.

∴ The area of the triangle is 20 square inches.

**Reasonable?** Draw the triangle on grid paper and count unit squares. Each square in the grid represents 1 square inch.



Squares full or nearly full: 18

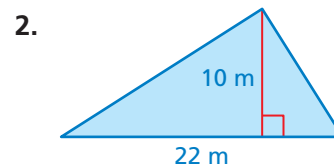
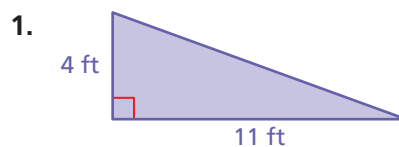
Squares about half full: 4

The area is  $18(1) + 4\left(\frac{1}{2}\right) = 20$  square inches.

So, the answer is reasonable. ✓

## On Your Own

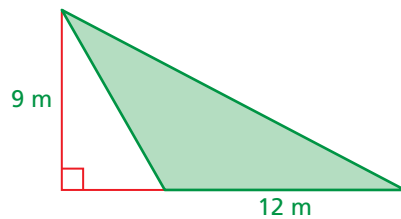
Find the area of the triangle.



Now You're Ready  
Exercises 3–8

## EXAMPLE 2 Finding the Area of a Triangle

Find the area of the triangle.

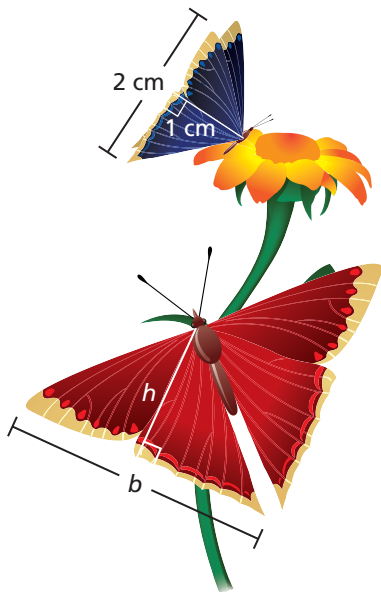


$$\begin{aligned} A &= \frac{1}{2}bh && \text{Write formula.} \\ &= \frac{1}{2}(12)(9) && \text{Substitute 12 for } b \text{ and 9 for } h. \\ &= 54 && \text{Multiply.} \end{aligned}$$

∴ The area of the triangle is 54 square meters.

## EXAMPLE 3 Real-Life Application

The base and height of the red butterfly wing are two times greater than the base and height of the blue butterfly wing. How many times greater is the area of the red wing than the area of the blue wing?



Find the area of the blue wing.

$$\begin{aligned} A &= \frac{1}{2}bh && \text{Write formula.} \\ &= \frac{1}{2}(2)(1) && \text{Substitute 2 for } b \text{ and 1 for } h. \\ &= 1 \text{ cm}^2 && \text{Multiply.} \end{aligned}$$

The red wing dimensions are 2 times greater, so the base is  $2 \times 2 = 4$  cm and the height is  $2 \times 1 = 2$  cm. Find the area of the red wing.

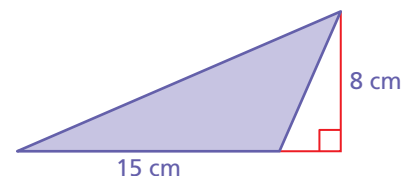
$$\begin{aligned} A &= \frac{1}{2}bh && \text{Write formula.} \\ &= \frac{1}{2}(4)(2) && \text{Substitute 4 for } b \text{ and 2 for } h. \\ &= 4 \text{ cm}^2 && \text{Multiply.} \end{aligned}$$

∴ Because  $\frac{4 \text{ cm}^2}{1 \text{ cm}^2} = 4$ , the area of the red wing is 4 times greater.

### On Your Own

Now You're Ready  
Exercises 12–14

- Find the area of the triangle.
- WHAT IF?** In Example 3, the base and the height of the red butterfly wing are three times greater than those of the blue wing. How many times greater is the area of the red wing?



## Vocabulary and Concept Check

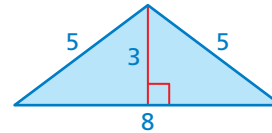
- CRITICAL THINKING** Can *any* side of a triangle be labeled as its base? Explain.
- DIFFERENT WORDS, SAME QUESTION** Which is different? Find “both” answers.

What is the area of the triangle?

What is the distance around the triangle?

How many unit squares fit in the triangle?

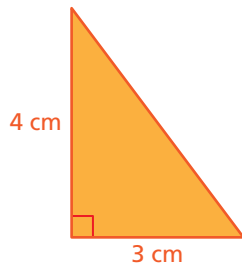
What is one-half the product of the base and the height?



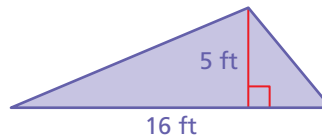
## Practice and Problem Solving

Find the area of the triangle.

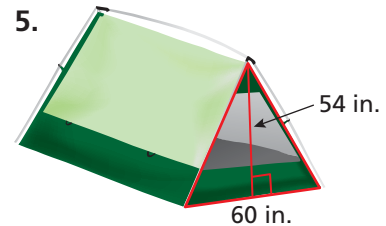
1 3.



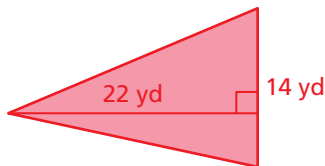
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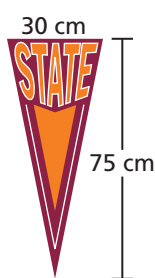
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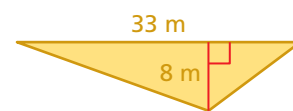
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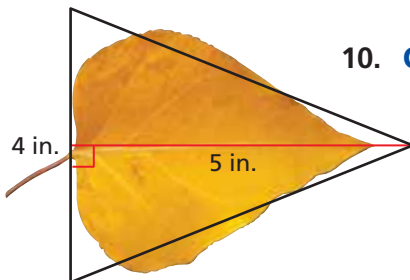


9. **ERROR ANALYSIS** Describe and correct the error in finding the area of the triangle.

**X**

$$A = \frac{1}{2}(10)(13) = 65 \text{ m}^2$$

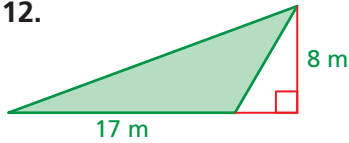
10. **COTTONWOOD LEAF** Estimate the area of the cottonwood leaf.



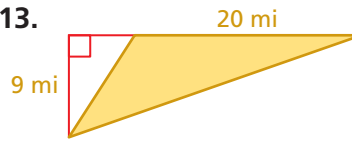
11. **CORNER SHELF** A shelf has the shape of a triangle. The base of the shelf is 36 centimeters, and the height is 18 centimeters. Find the area of the shelf.

Find the area of the triangle.

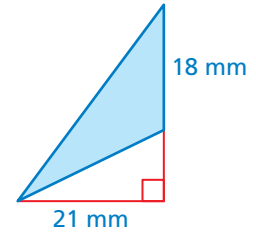
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13.



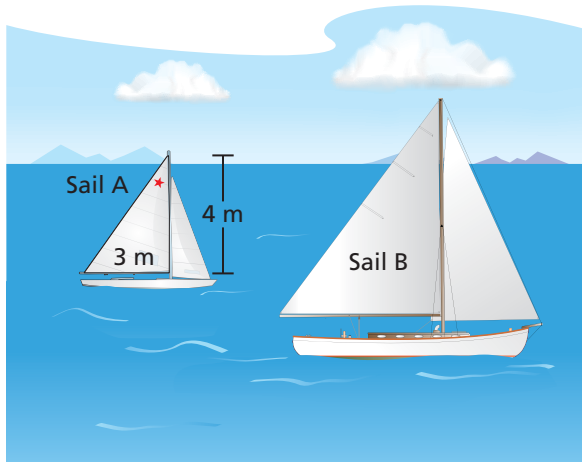
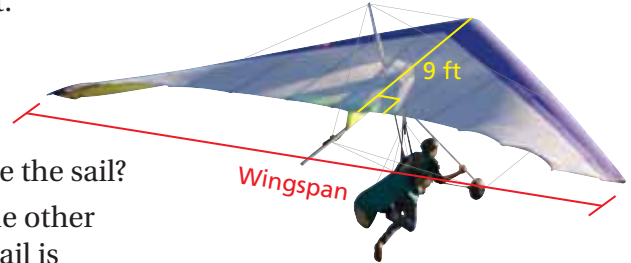
14.



15. **OPEN-ENDED** Draw and label two triangles that each have an area of 24 square feet.

16. **HANG GLIDING** The wingspan of the triangular hang glider is 30 feet.

- How much fabric is needed to make the sail?
- RESEARCH** Use the Internet or some other source to find how the area of the sail is related to the weight limit of the pilot.

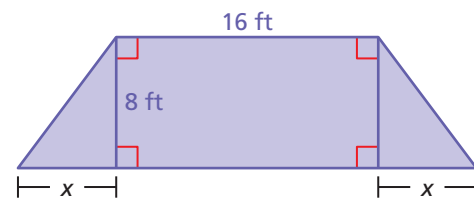


17. **SAILBOATS** The base and the height of Sail B are  $x$  times greater than the base and the height of Sail A. How many times greater is the area of Sail B? Write your answer as a power.

18. **WRITING** You know the height and the perimeter of an equilateral triangle. Explain how to find the area of the triangle. Draw a diagram to support your reasoning.

19. **REASONING** The base and the height of Triangle A are half the base and the height of Triangle B. How many times greater is the area of Triangle B?

20. **Critical Thinking** The total area of the polygon is 176 square feet. Find the value of  $x$ .



## Fair Game Review what you learned in previous grades & lessons

Tell which property is illustrated by the statement. (Section 3.3)

21.  $n \cdot 1 = n$

22.  $4 \cdot m = m \cdot 4$

23.  $(x + 2) + 5 = x + (2 + 5)$

24. **MULTIPLE CHOICE** What is the first step when using order of operations? (Section 1.3)

- (A) Multiply or divide from left to right. (B) Add or subtract from left to right.  
 (C) Perform operations in parentheses. (D) Evaluate numbers with exponents.